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Paschos-Wolfenstein relationship for nuclei and the NuTeV $\sin^2\theta_W$ measurement

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We discuss the nuclear effects in the Paschos-Wolfenstein relationship in the context of the extraction of the weak mixing angle. We point out that the neutron excess correction to the Paschos-Wolfenstein relationship for a neutron-rich target is negative and large on the scale of the experimental errors of a recent NuTeV measurement. We find a larger neutron excess correction to the Paschos-Wolfenstein relationship for the total cross sections than that discussed by the NuTeV Collaboration. The phenomenological applications of this observation are discussed in the context of the NuTeV deviation. The uncertainties in the neutron excess correction are estimated. The effects due to the Fermi motion, nuclear binding, and nuclear shadowing are also discussed in the context of the total cross sections.

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The NuTeV Collaboration recently reported the results of the measurement of the weak mixing angle in deep-inelastic neutrino and antineutrino scattering from a heavy target [1]. The NuTeV value of $\sin^2\theta_W = 0.2277 \pm 0.0013(\text{stat.}) \pm 0.0009(\text{syst.})$ turned out to be significantly larger than that derived from a global standard model fit to other electroweak measurements, 0.2227 ± 0.0004 [2]. The discussion of the possible uncertainties and physics behind this discrepancy can be found in Ref. [3]. However, before speculating on the possible new physics, one should first worry about the "standard" effects and uncertainties.

A useful tool, employed by the NuTeV Collaboration to derive $\sin^2\theta_W$, is the Paschos-Wolfenstein (PW) relationship [4]

$$R^- = \frac{\sigma^{\nu}_{\rm NC} - \sigma^{\bar\nu}_{\rm NC}}{\sigma^{\nu}_{\rm CC} - \sigma^{\bar\nu}_{\rm CC}} = \frac{1}{2} - \sin^2\theta_W, \tag{1}$$

where $\sigma^{\nu}_{\rm NC}$ and $\sigma^{\nu}_{\rm CC}$ are the deep-inelastic neutrino cross sections for the neutral current (NC) and charged-current (CC) interactions, and $\sigma^{\bar\nu}_{\rm NC}$ and $\sigma^{\bar\nu}_{\rm CC}$ are the corresponding antineutrino cross sections [5].

In a world without heavy quarks and with an exact isospin symmetry, Eq. (1) is an exact relation for the isospin zero target for both the total and differential cross sections. The validity of this relationship is solely based on the isospin symmetry [4]. Therefore, Eq. (1) also holds for a nuclear target, provided that the nucleus is in the isoscalar state. In particular, this means that various strong interaction effects, including the nuclear effects, must cancel out in ratio (1). However, in the real world Eq. (1) must be corrected for the *s*- and *c*-quark effects. Furthermore, the targets used in the neutrino experiments are usually nonisoscalar nuclei, and Eq. (1) must also be corrected for the nonisoscalarity effects, as well as for other nuclear effects such as the Fermi motion, binding, and nuclear shadowing.

The nuclear effects in the context of $\sin^2\theta_W$ were recently discussed in Refs. [6–8]. It was suggested in Ref. [6] that a difference between the nuclear shadowing in the NC and CC interactions may account for the discrepancy in NuTeV's measurement of $\sin^2\theta_W$, although no specific calculation of this effect was given. The impact of the nuclear effects on the extraction of $\sin^2\theta_W$ from data was discussed in Ref. [7] in terms of a phenomenological parametrization of the nuclear effects in the parton distributions derived from the charged lepton deep-inelastic scattering. The phenomenological studies of the difference between the nuclear effects in the *u*- and *d*-quark distributions in the context of the PW relationship were presented in Ref. [8].

In this paper we address the nuclear effects in the PW ratio for the total cross sections. We start from the discussion of the nonisoscalarity correction to the PW ratio, since there appears to be some confusion about the sign and the magnitude of this effect in the literature. Then we discuss the Fermi motion, nuclear binding, and nuclear shadowing corrections to the PW ratio.

I. THE PW RELATIONSHIP FOR GENERIC TARGET

We discuss the (anti)neutrino deep-inelastic scattering in the leading twist QCD approximation assuming that the four-momentum transfer Q is large enough. In this approximation the NC and CC structure functions are given by the well-known expressions in terms of the quark and antiquark distribution functions [9]. In order to simplify the discussion of the isospin effects, we consider the isoscalar, $q_0(x) = u(x) + d(x)$, and isovector, $q_1(x) = u(x) - d(x)$, quark distributions (for simplicity, we suppress the explicit notation for the Q^2 dependence of the parton distributions). The calculation of the NC and CC cross sections, and the PW ratio in the QCD parton model is straightforward. A QCD radiative correction to the PW ratio for the total cross sections was calculated in Ref. [3]. The result can be written as

$$R^- = \frac{1}{2} - s_W^2 + \left[1 - \frac{7}{3}s_W^2 + \frac{4\alpha_s}{9\pi}\left(\frac{1}{2} - s_W^2\right)\right]\left(\frac{x_1^-}{x_0^-} - \frac{x_s^-}{x_0^-} + \frac{x_c^-}{x_0^-}\right), \tag{2}$$

where $s_W^2 = \sin^2\theta_W$ [10], α_s is the strong coupling, and $x_a^- = \int dx\, x(q_a - \bar q_a)$, with q_a and $\bar q_a$ the distribution functions

*Electronic address: kulagin@ms2.inr.ac.ru of quarks and antiquarks of type a [11]. The subscripts 0 and 1 refer to the isoscalar q_0 and isovector q_1 quark distributions, respectively. In the derivation of Eq. (2) we expanded in x_1^-/x_0^- and $x_{s,c}^-/x_0^-$ and retained only the linear corrections. Equation (2) applies to any (not necessarily isoscalar) nuclear target. We observe that the PW relationship (1) is corrected by the C-odd parts of the isovector component in the target (x_1^-) and the strange and charm components of the target's sea.

The isovector quark distribution q_1 vanishes in an isoscalar target, provided that the isospin symmetry is exact. However, a correction due to the quark-antiquark asymmetry in the nucleon (or nuclear) strange sea is possible. In particular, a positive x_s^- would move s_W^2 towards a standard model value [3]. However, the available phenomenological estimates are controversial even in the sign of the effect: a shift in s_W^2, estimated in Ref. [3], is -0.0026, however, the NuTeV Collaboration gives a positive shift of 0.0020 ± 0.0009 [12].

II. THE NONISOSCALARITY CORRECTION IN A HEAVY TARGET

Heavy nuclei, such as iron, have unequal number of neutrons (N) and protons (Z). The ground state of such nuclei is not the isospin zero state but rather a mixture of different isospins. Therefore, the isovector quark distribution is finite in such nuclei that causes a finite non-isoscalarity correction to the structure functions and cross sections. This correction turns out to be large in the context of the NuTeV experiment, and, therefore, must carefully be taken into account.

In order to understand this effect, we first neglect other nuclear effects and view the neutrino scattering off a nucleus as incoherent scattering off bound protons and neutrons at rest. In this approximation, the nuclear distribution of partons of type a is the sum of those for bound protons and neutrons,

$$q_{a/\mathrm{A}}=Zq_{a/\mathrm{p}}+Nq_{a/\mathrm{n}}=\frac{A}{2}(q_{a/\mathrm{p}}+q_{a/\mathrm{n}})+\frac{Z-N}{2}(q_{a/\mathrm{p}}-q_{a/\mathrm{n}}), \tag{3}$$

where $A=Z+N$. We now apply this relation to the isoscalar and isovector quark distributions. Assuming an exact isospin symmetry, we have $q_{0/\mathrm{p}}=q_{0/\mathrm{n}}$ and $q_{1/\mathrm{p}}=-q_{1/\mathrm{n}}$. For the nuclear parton distributions per one nucleon we then obtain

$$A^{-1}q_{0/\mathrm{A}}=q_{0/\mathrm{p}}, \tag{4a}$$

$$A^{-1}q_{1/\mathrm{A}}=-\delta N q_{1/\mathrm{p}}, \tag{4b}$$

where we introduced a fractional excess of neutrons $\delta N=(N-Z)/A$. Similar equations can readily be written for the antiquark distributions.

Equations (4) suggest a *negative* neutron excess correction to R^- in a neutron-rich target. Indeed, using Eqs. (4), we find from Eq. (2) that the correction is

$$\delta R^-=-\delta N\frac{x_1^-}{x_0^-}\left[1-\frac{7}{3}s_W^2+\frac{4\alpha_s}{9\pi}\left(\frac{1}{2}-s_W^2\right)\right], \tag{5}$$

where x_1^- and x_0^- are taken for the proton.

The NuTeV Collaboration takes into account the nonisoscalarity correction in the analysis and discusses this effect on R^- [13,12]. However, Eq. (5) is different from the corresponding NuTeV equation (Eq. (7) of Ref. [12] and Eq. (9) of Ref. [13]). First, it must be noted that the NuTeV equation has the wrong sign of the δN term. Nevertheless, the NuTeV nonisoscalarity correction is eventually negative [14]. Therefore, in the following discussion we assume a correct sign in Eq. (7) in Ref. [12]. Second, Eq. (2) involves only C-odd terms, hence the factor x_1^-/x_0^-. The corresponding factor in Refs. [13,12] is $(x_u-x_d)/(x_u+x_d)$, where $x_a=\int dx x q_a(x)$ for the quark distribution q_a in the proton. This factor lacks the contribution from antiquarks and has a mixed C parity. For this reason it is not allowed in Eq. (5). Furthermore, Eq. (5) includes the α_s correction, which was not taken into account in Refs. [13,12].

In order to understand the magnitude of the nonisoscalarity correction to R^- and compare it to that in Refs. [13,12], we first neglect the α_s correction and compute $x_1^-/x_0^-=0.43$ and $(x_u-x_d)/(x_u+x_d)=0.34$ using the parton distributions of Ref. [15] (CTEQ5) in the modified minimal subtraction scheme evaluated at $Q^2=20$ GeV^2, an average Q^2 in the NuTeV experiment. For the neutron excess, we use the value $\delta N=0.0567$ reported by NuTeV [12]. Using $s_W^2\simeq 0.22$ and collecting all factors in Eq. (5), we obtain the neutron excess correction to R^- about -0.012. The corresponding correction computed using Eq. (7) in Ref. [12] is -0.0094. This correction is smaller in magnitude because the factor $(x_u-x_d)/(x_u+x_d)$ is smaller than the factor x_1^-/x_0^- by about 25%. In order to illustrate the sensitivity of $\sin^2\theta_W$ to the treatment of the neutron excess correction, we apply the difference $0.0094-0.012=-0.0026$ to the NuTeV central value of $\sin^2\theta_W$. Then we get a smaller value 0.2251, which is now about 1.5σ away from the standard model value.

The NuTeV Collaboration reported the neutron excess correction to be -0.0080 [13,14]. Although this correction was discussed in Ref. [13] in the context of R^-, it does not directly apply to the ratio of the total cross sections. Zeller and McFarland explain that this is the shift in the NuTeV $\sin^2\theta_W$ derived from a Monte Carlo simulation which contains the neutrino flux as well as all the experimental effects of cuts, smearing, resolutions, etc. [16].

The α_s correction to R^- is also negative, but small compared to the leading term. Using the next-to-leading order (NLO) $\alpha_s=0.2161$ at $Q^2=20$ GeV^2, we find the α_s correction to R^- about -0.0002.

Let us now discuss the possible uncertainties in the nonisoscalarity correction. Using Eq. (5) and the error in the neutron excess in the NuTeV target of 0.02% [13], we obtain the uncertainty in $\sin^2\theta_W$ of 0.00004. However, this is not the only source of uncertainties in the δN correction. Indeed, the ratio x_1^-/x_0^- is subject to the theoretical and experimental uncertainties. This ratio depends on the set of the parton distributions as well as on the order of perturbation theory to which the analysis is performed [17]. For example, the ratio x_1^-/x_0^- is very similar for the CTEQ5 and the Martin-Roberts-Stirling-Thorne parton distributions [18]. However, it is somewhat larger, $x_1^-/x_0^-=0.445$, for Alekhin's parton distributions [19]. Furthermore, the parton distributions are known with some error. This error was estimated in a recent analysis of Ref. [19], in which the experimental errors in deep-inelastic data as well as a number of theoretical uncertainties were taken into account. Using the results of Ref. [19], we have the estimation on the uncertainty in the ratio $\delta(x_1^-/x_0^-)\simeq 0.04$.[1] This gives the uncertainty in R^- about 0.001, much larger than the uncertainty due to the error in the neutron excess in the target. The NuTeV Collaboration takes into account similar uncertainty in the analysis, i.e., the uncertainty in the d/u ratio of the quark distributions [14]. However, the reported uncertainty in the NuTeV $\sin^2\theta_W$ is much smaller, 0.00005 [1]. We also comment that the variations of the parton distributions with Q^2 introduce additional uncertainty, which is hard to access in the present analysis.

III. THE FERMI MOTION AND NUCLEAR BINDING CORRECTIONS

We briefly discuss other nuclear effects on R^-. In order to sort out different effects, in the present discussion we do not consider explicit violations of the isospin symmetry. We recall that the isospin symmetry requires the nuclear effects to cancel out in the R^- ratio for the isoscalar target. For a nonisoscalar nucleus, Eq. (5) was derived neglecting the effects of the Fermi motion and nuclear binding. However, it is rather obvious that these effects and the nonisoscalarity correction must be considered in a unified approach. In such an approach Eq. (3) must be replaced by a convolution of the quark distributions with the nuclear distributions of bound protons and neutrons [20]. Note that in the approximation in which the proton and neutron distributions are identical, the Fermi motion and nuclear binding effects cancel out in the ratio x_1^-/x_0^- [21]. Therefore, a possible correction comes through the difference between the proton and neutron distribution functions and is likely to be small. One source of the correction is a nonlinear dependence of the nucleon distribution functions on the number of bound particles. In heavy nuclei with $Z\neq N$ this effect results in different nucleon distribution functions in the isovector and isoscalar channels even if no violation of the isospin symmetry is admitted [22]. Because of this effect the ratio x_1^-/x_0^- receives a correction factor of $1-\frac{2}{9}T/M$, where T is the average kinetic energy of the bound nucleon and M is the nucleon mass [23]. For the iron nucleus this factor is 0.993 that gives only a small correction to R^-.

IV. THE NUCLEAR SHADOWING EFFECT

We now discuss the nuclear shadowing effect in the context of the isoscalar and isovector quark distributions. The nuclear shadowing corrections to the quark distributions can be written as

$$A^{-1}q_{0/A}=q_{0/p}+\delta_{sh}q_0, \tag{6a}$$

$$A^{-1}q_{1/A}=-\delta N q_{1/p}+\delta_{sh}q_1. \tag{6b}$$

Similar equations can be written for the antiquark distributions. The nuclear shadowing effect in the isoscalar C-even ($q_0^+=q_0+\overline{q_0}$) and C-odd ($q_0^-=q_0-\overline{q_0}$) quark distributions was studied in Ref. [24], where a different magnitude of the shadowing effect in q_0^+ and in q_0^- was observed. In particular, it was argued that the relative shadowing effect in the C-odd quark distribution is enhanced compared to that in the C-even distribution (see also [25]). In order to estimate the shadowing effect in the total cross sections, we apply the approach of Ref. [24] and calculate the shadowing corrections to the average quark light-cone momentum, $\delta_{sh}x_0$, in the C-even and C-odd distributions. For the iron nucleus the results are $\delta_{sh}x_0^+/x_0^+=-0.01$ and $\delta_{sh}x_0^-/x_0^-=-0.002$. We observe that the shadowing effect in the total cross sections is significantly bigger in the C-even channel, in spite of the enhancement of the relative nuclear shadowing effect in the C-odd quark distributions. This, paradoxical from the first glance, result can be understood by observing that the C-odd cross section is saturated by the valence region at large x, where the nuclear shadowing effect is small. On the other hand, a large part of the C-even cross section comes from the sea region, where the nuclear shadowing effect is essential.

Since the isovector distribution must vanish in the isoscalar nucleus, the shadowing effect in the isovector channel appears at least in the order of δN. This effect has not been quantitatively studied yet. In our estimates we neglect the shadowing correction to x_1^-, leaving this interesting question to further studies. Then, the nuclear shadowing correction reduces to the renormalization of the ratio x_1^-/x_0^- by the factor of 1.002 for the iron nucleus.

Combining the nuclear shadowing effect with the Fermi motion and nuclear binding corrections to R^-, we observe a partial cancellation between these effects. For the iron nucleus the resulting correction factor to the ratio x_1^-/x_0^- is 0.995 that gives a negligible correction to R^-.

V. SUMMARY

We have studied the nuclear effects in the PW relationship for the total cross sections. We observed that for a neutron-rich target (such as iron, used in the NuTeV experiment) the neutron excess correction to the PW relationship is negative and large on the scale of the experimental errors. We found a larger neutron excess correction than that discussed by the NuTeV Collaboration in the context of R^-. The ratio R^- is rather sensitive to the value of the neutron excess correction and a consistent treatment of this correction may well explain at least a large part of the NuTeV deviation.

The uncertainties in the nuclear nonisoscalarity correction were discussed. We found that this uncertainty is dominated by the uncertainty in the parton distributions leading to the variation in R^- about 0.001 that should have a significant

[1] I am grateful to S. Alekhin for providing this estimate.

impact on the theoretical error in $\sin^2\theta_W$.

We discussed the effects of the Fermi motion, nuclear binding, and nuclear shadowing on the ratio R^- for the total cross sections. We point out that the nuclear effects on R^- vanish for the isoscalar nucleus and appear in the first or higher order in δN. For this reason these effects are small. Furthermore, we observed a partial cancellation between the Fermi motion and the nuclear shadowing effects in the ratio R^-.

However, this cancellation of the nuclear effects in the ratio R^- for the total cross sections does not, of course, mean that these effects identically cancel out in the ratio of the differential cross sections. Since the NuTeV experiment does not measure the total cross sections, some nuclear effects which cancel out in R^- remain in actual experimental observables. Therefore, in order to clarify the impact of the nuclear effects on the NuTeV result, it is necessary to explicitly take them into account in the analysis.

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